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Book Reviews

Books

H. STAHL AND V. TOTIK, *General Orthogonal Polynomials*, Encyclopedia of Mathematics and Its Applications, Vol. 43, Cambridge University Press, 1992, xii + 250 pp.

In the present book the authors consider general orthogonal polynomials in the sense that the orthogonality is with respect to a positive Borel measure μ in the complex plane. The emphasis is on the asymptotic behavior of $|p_n(z; \mu)|^{1/n}$ as $n \to \infty$. Such asymptotic behavior is relevant for finding the domain of convergence of Fourier series in orthogonal polynomials. Taylor series converge in disks whereas Legendre (Jacobi) series converge in elliptic domains. Under reasonable assumptions on μ and its support $S(\mu)$ one expects convergence of the Fourier series in a domain of the form $\{g_\Omega(z) < C\}$, where $g_\Omega(z)$ is the Green function of the unbounded component Ω of the support $S(\mu)$. For this nth root asymptotic behavior the natural theoretical setting is (logarithmic) potential theory and this book is a great demonstration of mathematical analysis in general and potential theory in particular. Already on page 4 we find one of the most beautiful results of the last few years, namely the upper and lower bounds for $|p_n(z; \mu)|^{1/n}$. The lower bound $e^{g_\Omega(z)}$ is indeed in terms of the Green function for the outer component Ω of the support $S(\mu)$, but in general a possibly larger upper bound $e^{g_\mu(z)}$ is valid, which is related to the carriers of μ. Carriers are Borel sets B such that $\mu(B) = \mu(S(\mu))$. The support is a carrier, but quite often it is too big for various estimates. Carriers give more detail on how the mass of μ is distributed over its support.

The first two chapters deal with general properties of orthogonal polynomials, bounds, norms, and inequalities in terms of capacities of supports and carriers. Chapters 3, 4, and 5 deal with the class of regular measures, i.e., measures μ such that the nth root asymptotic behavior is determined by the equilibrium measure of the support and the Green function of the unbounded component of the support. In Chapter 4, for instance, various sufficient conditions are given for a measure to be regular, and the relation between known conditions by Erdős-Turán, Ullman, and Widom is clarified. Chapter 6 deals with seven applications such as rational interpolation to Markov functions, best rational approximation to Markov functions with a remarkable new result on the exact rate of convergence, convergence of non-diagonal Padé approximants to Markov functions, and weighted polynomials in L^p.

This is an important but difficult book, necessary to anybody interested in advanced features of complex approximation theory, orthogonal polynomials, measure theory, and logarithmic potential theory. An introduction in (logarithmic) potential theory is recommended and readers are advised to consult, at an early stage, the appendix in which a brief survey is given of the most relevant aspects of potential theory.

ALPHONSE MAGNUS AND WALTER VAN ASSCHE

L. LORENTZEN AND H. WAADELAND, *Continued Fractions with Applications*, Studies in Computational Mathematics, Vol. 3, North-Holland, 1992, xvi + 606 pp.

Continued fractions have always played an important role in mathematics, sometimes in disguise as for instance in the theory of difference equations (although one could always argue that continued fractions are but another face of this theory). They have always acted as a binding factor between diverse subjects such as orthogonal polynomials, geometry of zeros,

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